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Effect of Advance Payment and Trade Credit Policy on Supply Chain Inventory model for instantaneous deteriorating items under Inflation for Post Covid-19 Recovery

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Abstract: The behavior, capital, priority, and payment policies for supply chain players have been affected by the COVID-19 pandemic disaster. This research paper develops a mathematical model in which advanced and delayed payment policies are planned with price- and time-sensitively dependent demand under inflation for the post-COVID-19 recovery. This model also takes into account time-dependent deterioration, shortages, instantaneously deteriorating items, and partial backlogging. With the use of this model, retailers can reduce overall costs in various payment scenarios by determining the best replenishment cycle and order quantity. To keep orders moving from customers to retailers and retailers to suppliers throughout the financial crisis, we designed a partially advanced and delayed payment policy. We look at how advanced and late payments affect a retailer's overall costs. The application of this model is demonstrated by numerical examples, which are solved by MATHEMATICA 12.0. An investigation into the sensitivity of crucial parameters has been carried out to identify additional sensitive parameters that offer a precise depiction of the current issues.

Keywords: Covid-19, Inventory, Advance and delay-in-payment, Deteriorating items, Inflation.

1. Introduction & Literature Review

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In today's growing business world, the supply chain is more important than ever. Increasing accountability and resilience will enable companies to rebuild or reuse supply chains in the event of future disruptions and to react quickly to current ones. Companies must overcome the economic and operational difficulties brought on by the Corona virus in order to swiftly satisfy the needs of their staff, customers, and suppliers. As consumers, workers, citizens, and people in general, the global COVID-19 pandemic has irreversibly changed our perspectives, actions, and life experiences. Consumer purchasing patterns are being drastically altered by the crisis, which is hastening important structural changes in industries that depend on consumer goods. Fundamental changes in consumer behavior, supply chains, and marketing channels have knocked firms off balance because of the COVID-19 issue.

In a pandemic, the retailer might assist its supplier by making payments in advance, according to Gupta and Chutani (2020). The three payment options are frequently used by buyers and sellers: trade credit or payment deferral, advance payment, and immediate payment. Trade promotion can benefit from these payment plans since they benefit both the customer and the supplier. The primary policy is an immediate payout that is similar to the popular EOQ model that Harris popularized (1990). According to the second policy, the goods must be paid for by the customer in advance of delivery. Depending on the seller, the buyer may be required to pay the entire purchase price or a portion of it in advance (Zhang *et al.* (2014) Lashgari *et al.* (2016)). Retailers accept trade credit or delays in payment under the third policy. The two components of the delayed payment policy are the complete payment and the partial payment. When a full payment delay is applied, all purchasing costs can be paid after the scheduled date, as opposed to a partial payment delay that only allows for payment of part of the overall purchase price at the time of delivery and the remaining amount at a later time. The customer and the retailer refrain from making all payments at once because of a financial problem with COVID-19. This study has taken into account this problem by utilizing a partial advance and payment delay system with price- and time-dependent demand and inflation.

A stock's utility or marginal value is lost due to deterioration, which can be caused by alterations, deterioration, damage, spoiling, degradation, and theft. Items with limited shelf life include medicine, blood, seafood, alcohol, etc. They all begin to lose their quality as soon as they are brought into stock. As a result, it is impossible to ignore the product's deterioration. It is a common misconception among inventory models that in situations of stock outs, all demand is lost or backlogged. Actually, some customers become agitated and leave while others wait for replenishment, particularly if there is a brief wait. This investigation has used partial backlogging in light of these stock-out scenarios.

1.1 Main contributions of the study include the following

1. This model examined two techniques and payment options in a pandemic scenario.
2. To support post-Covid-19 recovery, a hybrid payment strategy is being considered.

1.2 The literature overview on Advance payment

Making use of a discounted cash flow study, guidelines for lot-sizing and pricing of perishable goods with cash credit payment in advance were developed by R. Li. *et al.* (2017). Wu *et al.* (2018) provided policies on inventory for perishable products with expiration dates as well as advance-cash-credit payment plans. Khan *et al.* (2019) developed a two-warehouse inventory model for items that deteriorate with an advanced payment plan. With a constant partial backlog rate, shortages are permitted. Al-Amin Khan *et al.* (2020) provided an inventory model for items that deteriorate, taking stock and price dependence into account and allowing shortages under both partial and full advance payment scenarios. A sustainable inventory model based on advanced payment, deterioration, and controlled carbon emissions was created by Mashud *et al.* (2021). Shaikh *et al.* (2022) examined how COVID-19 affected the need for an inventory model under advance payment facilities and preservation technologies. In a partially backlogged inventory model with interval uncertainty, as shown by Mondal *et al.* (2023), products deteriorate at a specific rate when they are stock-in. Depending on the advanced payment and discount availability, two different scenarios are taken into consideration. Using partially advanced payment methods and preservation technologies, an EOQ inventory model has been developed by Sharma and Mondal (2024) for retailers who sell deteriorating items.

1.3 The literature overview on payment delay

A late payment policy was used in an inventory model that was developed by Liao *et al.* (2000). An EPQ inventory model for optimal cycle time was created by Chung *et al.* (2003) under trade credit policy. Liang *et al.* (2011) provided two warehouse inventory models for the products that degrade under a conditional trade credit policy. A two-warehouse mathematical model for degrading products with partial backlogging and late payment was considered by Bhunia *et al.* (2014) in the inventory system. A deteriorating item inventory control model was presented by Sen *et al.* [37] with shortage and time-dependent holding costs under payment terms that are acceptable for non-instantaneous deteriorating products. Mathur *et al.* (2019) examined a mathematical model of inventory with demand, which is variable, and the allowable payment delay. Two warehouse mathematical model of inventory was given by Chandra (2020) with items that deteriorate with stock-dependent demand under allowable payment delays. A fully acceptable payment delay policy at different intervals of trade credit was accepted in the generalized order-level inventory system by Mondal *et al.* (2021) An inventory model in which a discount on price was considered for deteriorating commodities under advance and late payment schemes was developed by Duary *et al.* (2022). Kaushik (2023) suggested an inventory model with variable interest rates and a payment delay that is acceptable. A model for EOQ under the condition of allowable payment delays with permitted stock-out costs and lead times was presented by Jayanthi (2024).

1.4 The literature overview on Time and Price dependent demand

In the formation of the EMQ (Economic Manufacturing Quantity) model for the imperfect production process, Sarkar and his colleague (2018) used the selling price and time-dependent demand. An inventory model in a fuzzy environment can be created by Hossen et al. (2016). They took the system's impact of inflation into account for items that deteriorate with time and price-dependent demand. A model of inventory for time-dependent non-instantaneous deteriorating products was developed by Debata et al. (2017). One assumes a demand function that is both time-dependent and price-dependent. A quadratic varying time function of the product's freshness, selling price, shelf space occupied by displayed stock, and expiration date is assumed to drive market demand for fresh produce products, according to the model of inventory developed by Shah & Sen (2019). A mathematical model for inventory and demand, which was based on price and time under time-dependent deterioration and inflation, was created by Saha and Naik (2018). A novel lot-size inventory problem for goods whose demand pattern is influenced by price, frequency of advertising, and time was investigated by San-José. et al. (2021) Aarya *et al.* (2022) devised a production inventory model that has two storage options and a demand that is contingent on both price and time. Singh et al. (2022) formulated an inventory model considering price- and time-dependent demand, incorporating the combined effects of preservation technology investment and order cost reduction, under various carbon emission policy frameworks. Using price, time-dependent demand, and advertising for deteriorating items, which was non-instantaneous, Narang & De (2023) addressed an inventory system for imperfect production. A complete model was presented by Kumar et al. (2024) that takes into account the complications brought about by the multiple locations of storage, deterioration of products, and demand dependent on selling price and time. Singh et al. (2024) developed a time-dependent demand model to analyze a supply chain in which raw materials are provided by a supplier to a manufacturer, who then produces the finished goods. Kumar et al. (2024) developed a production-inventory model incorporating both time-dependent demand and time-dependent deterioration.

1.5 The literature overview on Inflation

Inflation is a significant factor in production management and the inventory system, even though decision-makers may find it challenging to come to certain conclusions. The effects of inflation must currently be taken into account because they cannot be ignored and have an impact on the inventory system.

The first EOQ model was created by Buzacott (1975), accounting for inflationary effects. Under inflationary conditions, the inventory decision policy was proposed by Bierman and Thomas (1977). A dynamic mathematical inventory model can be economically analysed by Hariga (1994) with non-stationary costs and demand. Adak *et al.* (2018) showed a model of inventory with variable demand for cost, availability, and dependability that deteriorates with inflation and takes payment delays into account. A time- and price-dependent model of inventory for commodities that deteriorate with inflation-affected shortages and demand was provided by Saha and his colleague [34]. In a SC with multiple retailers and a single vendor, Esmaeili *et al.* (2021) introduced a model of inventory for degrading goods under inflation. Handa *et al.* (2021) assumed how inflation affects the production inventory model used to sell price-based demand during shortages and deteriorating items over time. Hossen *et al.* (2016) created the deteriorating items inventory model that considered the system's inflation effect and the advertisement-dependent demand frequency. Singh et al. (2022) proposed an Economic Production Quantity (EPQ) model that accounts for the effects of inflation and allows partial backlogging. Using the retailer's returns and the inflation rate, Kharidar *et al.* (2023) looked into inventory control and pricing for goods that were perishable. Pal *et al.* (2024) looked into an inventory problem that was non-instantaneously degrading and had permitted for payment payments delay under inflation.

2. Assumptions and Notations

A list of the model's assumptions and notations is as follows

2.1 Assumptions

1. Infinite replenishment rate is considered.
2. Constant lead time is considered.
3. Infinite planning horizon is considered.
4. Price and Time dependent demand rate D = $(a-bp+ct)$ is considered.
5. Partially backlogged demands are those that have not yet been met. As consumers' waiting times $(T-t)$ get shorter, the proportion of backorders increases. The partial backlog rate can be expressed as $e^{-v(T-t)}$,here v represents the positive backlog parameter.
6. Advance payment in n installments is requested by the supplier . The retailer then obtains a loan at a specific interest rate from a financial institution. Throughout the paper, an identical approach to that used by Lashgari et al. [35] is then used to calculate the total cost of capital (per cycle) of pre-payments.

2.2 Notations

A	Ordering cost
a	Basic Demand
b	Slope of demand function
C	Purchase cost

C_1	Holding cost
C_2	Shortage cost
C_3	Lost sale cost
D	Annually demand
I_1 (t)	Level of Inventory at time t , $t \in [0, t_1]$
I_2 (t)	Level of Inventory at time t , $t \in [t_1, T]$
L	Lead time
n	The no. of instalments which are prepayment
M	Payment period delay
T	Replenishment Cycle
S,R	Maximum amount of stock and shortage for each cycle, respectively
t1	The time where stock is zero
v (> 0)	Backlogging variable
p	Selling price
θ	Rate of deterioration
I_e	Interest rate earned annually
I_{CC}	Interest rate on loans, annual
I_c	Interest rate charged annually
$CCCP$	Cyclic capital cost
PC	Overall cost of purchase
LSC	Lost sale cost
BC	Backorder cost
σ	Purchase price fraction that needs to be paid before delivery
Q	Order quantity
IE_i , $i = 1, 2$	Earned Interest
IC_i , $i = 1, 2$	Interest charged
TC_i , $i = 1, 2$	Total cost

3. Description of the problem

Based on the aforementioned supposition, a supplier will demand σ% of the entire cost from a retailer at the time of order placement, and the remaining (1−σ)% will be paid at the time of order receipt . A delay in payment may be offered by the retailer to its customer. The Product has a non-instantaneous deteriorating nature, and there is a hybrid payment method. Due to requirements and decay, the stock level depletes during [0, t_1], and subsequently, some requirements are backlogged as a result of the shortage. Based on the effects of the pandemic on the market, The retailer will decide on the cycle of replenishment and the time where inventory is zero. Suggested EOQ models are shown in Figure 3.1.

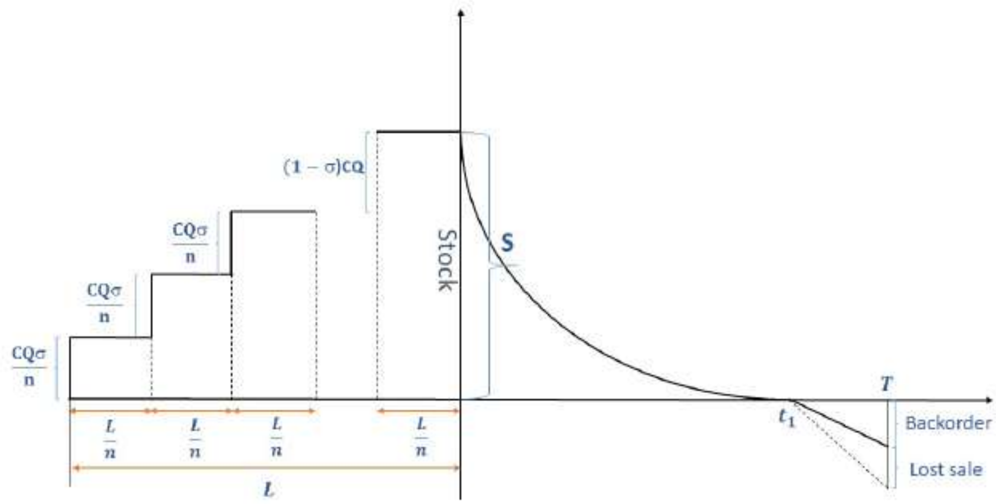

***Fig.* 3.1** *I(t) vs time with shortage*

This study offers two mathematical models for the system of hybrid payments that take shortages into account with consideration of two payment policies. 1.Delay in payment; and 2. Advanced payment . Here, deterioration and demand lead the level of inventory I (t) to zero at t = t_1, while shortages occur during the period [t_1, T] as a result of demand and some backlogged requirements. Figure 3.1's left portion shows that several prepayments were made within the lead-time L, each for an equal length of time $\frac{L}{n}$. The small portion indicates the amount which the retailer will have to pay when the products are delivered, while the large portion shows payments made in advance. Diff. Eq. for the recommended system of inventory is

$$\text{(3.1)} \qquad \frac{dI_1(t)}{dt} + \theta t I_1(t) = -(\text{a} - \text{bp} + \text{ct}), \quad t \in (0, t_1]$$

Taking $I_1(\text{t}_1) = 0$

We have from (1)

(3.2) $$I_1(t)=[(a-bp)(t_1-t)+\frac{c}{2}(t_1^{\,2}-t^2)+\frac{(a-bp)\theta}{6}(t_1^{\,3}-t^3)+\frac{c\theta}{8}(t_1^{\,4}-t^4)]e^{\frac{-\theta t^2}{2}}$$

With $I_1(0)=S$, Initial stock is provided by

(3.3) $$S=[(a-bp)\,t_1+\frac{c}{2}t_1^{\,2}+\frac{(a-bp)\theta\, t_1^{\,3}}{6}+\frac{c\theta}{8}t_1^{\,4}]$$

Following the time t = t_1, inventories progressively decline to zero, causing shortages. There is a partial backlog of requirements with a rate of (T −t) in the period of shortage [t_1, T]. Diff. Eq. for this can be written as

(3.4) $$\frac{dI_2(t)}{dt}=-(a-bp+ct)e^{-v(T-t)},\ t\in(t_1\,,T]$$

$$I_2(t_1)=0,$$

So

(3.5) $$I_2(t)=\left(\frac{a-bp}{v}\right)\left[e^{v(t_1-T)}-e^{v(t-T)}\right]+c\left[\left(\frac{t_1}{v}-\frac{1}{v^2}\right)e^{v(t_1-T)}-\left(\frac{t}{v}-\frac{1}{v^2}\right)e^{v(t-T)}\right],$$
$$t\in(t_1\,,T]$$

So, The following provides the max. shortage

(3.6) $$R=-I_2(T)=-\left[\left(\frac{a-bp}{v}\right)+c\left(\frac{t_1}{v}-\frac{1}{v^2}\right)\right]e^{v(t_1-T)}+\left[\frac{a-bp}{v}+c\left(\frac{T}{v}-\frac{1}{v^2}\right)\right]$$

When the total initial stock and total shortages are used to calculate the retailer's total order, the result takes on the following mathematical form

Q=S + R

(3.7) $$Q=(a-bp)t_1+\frac{c}{2}t_1^{\,2}+\frac{(a-bp)\theta}{6}t_1^{\,3}+\frac{c\theta}{8}t_1^{\,4}-\left[\frac{a-bp}{v}+c\left(\frac{t_1}{v}-\frac{1}{v^2}\right)\right]e^{v(t_1-T)}+\left[\frac{a-bp}{v}+c\left(\frac{T}{v}-\frac{1}{v^2}\right)\right]$$

The ordering cost of the system can be written as

(3.8) O.C. = A

For products, a supplier is paid by a retailer. He is also required to retain the inventory for a particular time. Consequently, the cost of holding is stated as ,

$$H.\mathrm{C.} = C_1 \int_0^{t_1} I_1(\mathrm{t}).e^{-rt}\, dt$$

$$(3.9) \quad \mathrm{H.C.} = C_1 \left\{ (a-bp)\frac{t_1^{\,2}}{2} + \left[(a-bp)r + 2c \right]\frac{t_1^{\,3}}{6} + \left[2\theta(\mathrm{a-bp}) - 3\,\mathrm{cr} \right]\frac{t_1^{\,4}}{24} + \left[4c\theta - 3r\theta(\mathrm{a-bp}) \right] \frac{t_1^{\,5}}{60} - \left[(\mathrm{a-bp})\theta^2 + 3\,\mathrm{cr}\,\theta \right]\frac{t_1^{\,6}}{72} - c\theta^2\,\frac{t_1^{\,7}}{84} \right\}$$

The overall purchase cost is represented by $P.C. = CQ$.

$$P.C. = CQ$$

$$(3.10) \quad \mathrm{P.C.} = C\Big[(\mathrm{a-bp})\,\mathrm{t}_1 + \frac{c}{2}t_1^{\,2} + (\mathrm{a}-bp)\frac{\theta\,\mathrm{t}_1^{\,3}}{6} + \frac{c\theta t_1^{\,4}}{8} - \left[\frac{a-bp}{v} + c\left(\frac{t_1}{v} - \frac{1}{v^2} \right) \right] e^{v(\mathrm{t}_1 - \mathrm{T})} + \left[\frac{a-bp}{v} + c\left(\frac{T}{v} - \frac{1}{v^2} \right) \right] \Big]$$

There will be some customers who must wait for the next lot due to supply challenges . This indicates some unmet demand with a backorder cost for the interval $t_1 < t \le T$. Thus, the backorder cost (B.C.) is

$$B.C. = C_2 \int_{t_1}^{T} -I_2(\mathrm{t})\, \mathrm{e}^{-\mathrm{rt}}\, dt$$

$$\text{(3.11)} \qquad B.C. = C_2 \begin{bmatrix} \dfrac{e^{-vt}}{v(v-\mathrm{r})}[(\mathrm{a}-\mathrm{bp})-c\left(\dfrac{2v-r}{v(v-\mathrm{r})}\right)]\left[e^{(v-\mathrm{r})\mathrm{T}}-e^{(v-\mathrm{r})t_1}\right] \\ +\dfrac{e^{v(\mathrm{t}_1-\mathrm{T})}}{vr}\left[(\mathrm{a}-\mathrm{bp})+\dfrac{c(v\,\mathrm{t}_1-1)}{v}\right]\left[e^{-rT}-e^{-rt_1}\right]+ \\ \dfrac{ce^{-vT}}{v(v-\mathrm{r})}\left[Te^{(v-\mathrm{r})\mathrm{T}}-t_1e^{(v-\mathrm{r})\mathrm{t}_1}\right] \end{bmatrix}$$

Popular product stock outs can lead to lost sales as missed opportunities, which is expressed mathematically as:

$$LSC = C_3\int_{t_1}^{T}(\mathrm{a}-\mathrm{bp}+\mathrm{ct})(1-\mathrm{e}^{-v(\mathrm{T}-\mathrm{t})})e^{-rt}dt$$

$$\text{(3.12)} \qquad L.S.C. = C_3\left\{\left[\frac{(\mathrm{a}-\mathrm{bp})}{r}+\frac{c}{r^2}\right](\mathrm{e}^{-\mathrm{rt}_1}-\mathrm{e}^{-\mathrm{rT}})+\left(\frac{a-bp}{v-r}-\frac{c}{(v-\mathrm{r})^2}\right).\right.$$
$$\left(e^{v(\mathrm{t}_1-\mathrm{T})-\mathrm{rt}_1}-e^{-rT}\right)+\frac{c}{r}(\mathrm{t}_1\mathrm{e}^{-\mathrm{rt}_1}-Te^{-rT})$$
$$\left.+\frac{ce^{-vT}}{(v-r)}\left(t_1e^{(v-\mathrm{r})\mathrm{t}_1}-Te^{(v-\mathrm{r})\mathrm{T}}\right)\right\}$$

A portion σ is paid before the items are collected, as shown in figure 3.1. Upon delivery, the balance that remains (1− σ) is anticipated to be paid. This leads to the following writing of the cycle capital cost:

$$\text{(3.13)} \qquad CCCP = \frac{C\sigma I_{cc}\,\mathrm{Q}(\mathrm{n}+1)\,\mathrm{L}}{2n}$$

The retailer gives customers an extended payment time period (M) after the receiving time in an effort to raise sales and reverse the unfavorable situation. Different circumstances with differing effects on revenue and expense arise from the different sizes of M situations, i.e., M < L. The next section goes over the details of two different inventory systems.

Case1 $0 < M < t_1$, The amount of interest earned (IE_1) is correlated with the interest rate I_e. Prior to the shortage (t_1), the retailer received compound interest; however, simple interest is paid during the shortage. IE_1 is taken as ,

$$IE_1 = (1-\sigma)\Big[pI_e \int_0^{t_1}\int_0^{t} (\mathrm{a-bp+ct})\, e^{-rt} du dt + \frac{pI_e}{v}\int_0^{t_1} (\mathrm{a-bp+ct})\, e^{-rt}(1-\mathrm{e}^{-v(\mathrm{T-t_1})})\,\mathrm{dt}\Big]$$

(3.14) $$IE_1 = (1-\sigma)\, pI_e \Big\{ [(a-bp)\left[\frac{1}{r^2} - \left(\frac{1}{r^2}+\frac{t_1}{r}\right)e^{-rt_1}\right] - c\left[\frac{t_1^{\,2} e^{-rt_1}}{r} + \frac{2}{r^2} t_1 e^{-rt_1} + \frac{2}{r^3}\left(e^{-rt_1}-1\right)\right]] - \left(\frac{1-e^{-v(\mathrm{T}-t_1)}}{v}\right)\left[\left(\frac{a-bp}{r}\right)\left(e^{-rt_1}-1\right) + c\left[\frac{t_1 e^{-rt_1}}{r} + \frac{\left(e^{-rt_1}-1\right)}{r^2}\right]\right]\Big\}$$

The interest that is earned on the total inventory up to the (1−σ) portion is this. Until M, the consumer is not required to make payment because of the reasonable delay. M is followed by a charge to the retailer for the available inventory. Thus, in relation to interest rate I_c, IC_1 on the whole inventory is stated in this way

$$IC_1 = CI_c \int_M^{t_1} e^{-rt} I_1(\mathrm{t})\,\mathrm{dt}$$

$$(3.15)\qquad IC_1 = CI_c\left\{\left(-M+\frac{rM^2}{2}+\frac{\theta M^3}{6}\right)(\mathrm{a-bp})\,\mathrm{t}_1+\right.$$

$$\left[\left(\frac{a-bp}{2}\right)-\left(\mathrm{M}-\frac{rM^2}{2}-\frac{\theta M^3}{6}\right)\frac{c}{2}\right]t_1^{\,2}$$

$$+\left[\frac{c}{3}-\frac{(\mathrm{a-bp})\,\mathrm{r}}{6}-\left(M-\frac{rM^2}{2}-\frac{\theta M^3}{6}\right)\right.$$

$$\left.\left(\frac{a-bp}{6}\theta\right)\right]t_1^{\,3}+\left[\frac{(a-bp)\theta}{12}-\frac{cr}{8}\right.$$

$$\left.-\left(M-\frac{rM^2}{2}-\frac{\theta M^3}{6}\right)\frac{c\theta}{8}\right]t_1^{\,4}+\left[\frac{c\theta}{15}-\frac{(\mathrm{a-bp})\theta\,\mathrm{r}}{20}\right]t_1^{\,5}$$

$$-\left[\frac{(\mathrm{a-bp})}{72}\theta^2+\frac{cr\theta}{24}\right]t_1^{\,6}-\frac{c\theta^2}{84}t_1^{\,7}+(a-bp)\frac{M^2}{2}+$$

$$\left[\frac{c}{2}-(\mathrm{a-bp})\,\mathrm{r}\right]\frac{M^3}{3}-\left[\frac{(\mathrm{a-bp})\theta}{3}+\frac{cr}{2}\right]\frac{M^4}{4}-$$

$$\left.\left[\frac{c\theta}{8}+\frac{(\mathrm{a-bp})\theta\,\mathrm{r}}{6}\right]\frac{M^5}{5}-\left[\frac{cr\theta}{8}+\frac{(\mathrm{a-bp})\theta^2}{12}\right]\frac{M^6}{6}-\frac{c\theta^2}{112}M^7\right\}$$

This shows the overall cost per cycle in the following manner

$$TC(\mathrm{t}_1,\mathrm{T})=\frac{1}{T}\left[O.C.+H.C.+B.C.+LSC+P.C.+CCCP-IE_1+IC_1\right]$$

$$(3.16)\qquad TC(\mathrm{t}_1,\mathrm{T})=\frac{1}{T}\left[A+C_1\left\{(\mathrm{a-bp})\frac{t_1^{\,2}}{2}+\left[(\mathrm{a-bp})\,\mathrm{r}+2\,\mathrm{c}\right]\frac{t_1^{\,3}}{6}\right.\right.$$

$$+\left[2\theta(\mathrm{a-bp})-3\,\mathrm{cr}\right]\frac{t_1^{\,4}}{24}+\left[4c\theta-3r\theta(\mathrm{a-bp})\right]\frac{t_1^{\,5}}{60}$$

$$\left.-\left[(\mathrm{a-bp})\theta^2+3cr\theta\right]\frac{t_1^{\,6}}{72}-\frac{c\theta^2 t_1^{\,7}}{84}\right\}+C_2\left[\frac{e^{-vt}}{v(\mathrm{v-r})}\right.$$

$$\left((a-bp)-\frac{c(2v-\mathrm{r})}{v(v-\mathrm{r})}\right)\left(e^{(v-\mathrm{r})\mathrm{T}}-e^{(v-\mathrm{r})\mathrm{t}_1}\right)$$

$$+\frac{e^{v(\mathrm{t}_1-\mathrm{T})}}{vr}\left((\mathrm{a-bp})+\frac{c(v\,\mathrm{t}_1-1)}{v}\right)\left(e^{-rT}-e^{-rt_1}\right)$$

$$+\frac{ce^{-vT}}{v(\mathrm{v-r})}\left[Te^{(v-\mathrm{r})\mathrm{T}}-\mathrm{t}_1\,e^{(v-\mathrm{r})\mathrm{t}_1}\right]\Bigg]+C_3\left\{\left(\frac{a-bp}{r}+\frac{c}{r^2}\right)\left(e^{-rt_1}-e^{-rT}\right)\right.$$

$$+\frac{c}{r}\left(t_1e^{-rt_1}-Te^{-rT}\right)+\frac{ce^{-vT}}{(v-\mathrm{r})}(t_1e^{(v-\mathrm{r})}\,\mathrm{t}_1$$

$$\left.-Te^{(v-\mathrm{r})\mathrm{T}})\right\}+C\left[(\mathrm{a-bp})\,\mathrm{t}_1+\frac{ct_1^{\;2}}{2}+\frac{(\mathrm{a-bp})\theta\,\mathrm{t}_1^{\;3}}{6}+\frac{\mathrm{c}\,\theta\,\mathrm{t}_1^{\;4}}{8}-\left[\frac{a-bp}{v}+c\left(\frac{t_1}{v}-\frac{1}{v^2}\right)\right]\right.$$

$$\left.e^{v(\mathrm{t}_1-\mathrm{T})}+\left[\frac{a-bp}{v}+c\left(\frac{T}{v}-\frac{1}{v^2}\right)\right]\right]$$

$$+\frac{c\sigma IccQ(\mathrm{n+1})\,\mathrm{L}}{2n}-(1-\sigma)\,\mathrm{pI}_e\left\{(\mathrm{a-bp})\right.$$

$$\left[\frac{1}{r^2}-\left(\frac{1}{r^2}+\frac{t_1}{r}\right)e^{-rt_1}\right]-c\left[\frac{t_1^{\;2}e^{-rt_1}}{r}+\frac{2}{r^2}t_1e^{-rt_1}+\frac{2}{r^3}\left(e^{-rt_1}-1\right)\right]\Bigg]-\left(\frac{1-e^{-v(\mathrm{T}-\mathrm{t}_1)}}{v}\right)$$

$$\left[\left(\frac{a-bp}{r}\right)\left(e^{-rt_1}-1\right)+c\left[\frac{t_1e^{-rt_1}}{r}+\frac{\left(e^{-rt_1}-1\right)}{r^2}\right]\right]$$

$$\left.\right\}+CI_c\left\{\left(-M+\frac{rM^2}{2}+\frac{\theta M^3}{6}\right)(\mathrm{a-bp})\,\mathrm{t}_1\right.$$

$$+\left[\left(\frac{a-bp}{2}\right)-\left(M-\frac{rM^2}{2}-\frac{\theta M^3}{6}\right)\frac{c}{2}\right]t_1^{\;2}$$

$$+\left[\frac{c}{3}-\frac{(\mathrm{a-bp})\,\mathrm{r}}{6}-\left(M-\frac{rM^2}{2}-\frac{\theta M^3}{6}\right)\left(\frac{a-bp}{6}\right)\theta\right]t_1^{\;3}$$

$$+\left[\frac{(\mathrm{a-bp})\theta}{12}-\frac{cr}{8}-\left(M-\frac{rM^2}{2}-\frac{\theta M^3}{6}\right)\frac{c\theta}{8}\right]t_1^{\;4}$$

$$+\left[\frac{c\theta}{15}-\frac{(\mathrm{a-bp})\,\mathrm{r}\,\theta}{20}\right]t_1^{\;5}-\left[\frac{\left(a-bp\right)\theta^2}{72}+\frac{cr\theta}{24}\right]t_1^{\;6}$$

$$-\frac{c\theta^2}{84}t_1^{\;7}+\frac{(\mathrm{a-bp})\,\mathrm{M}^2}{2}+\left[\frac{c}{2}-(\mathrm{a-bp})\,\mathrm{r}\right]\frac{M^3}{3}$$

$$-\left[\frac{(a-bp)\theta}{3}+\frac{cr}{2}\right]\frac{M^4}{4}-\left[\frac{c\theta}{8}+\frac{(\mathrm{a-bp})\theta\,\mathrm{r}}{6}\right]\frac{M^5}{5}$$

$$-\left[\frac{cr\theta}{8}+\frac{(\mathrm{a-bp})\theta^2}{12}\right]\frac{M^6}{6}-\frac{c\theta^2}{112}M^7\Bigg\}\Bigg]$$

Case 2 $t_1 < M < T$,

For this case we have

$IE_2 = IE_1$

(3.17) $$=(1-\sigma)\,pIe\Bigg\{\Bigg[\left[(a-bp)\left[\frac{1}{r^2}-\left(\frac{1}{r^2}+\frac{t_1}{r}\right)e^{-rt_1}\right]\right.$$

$$-c\left[\frac{t_1^{\,2}e^{-rt_1}}{r}+\frac{2}{r^2}t_1e^{-rt_1}+\frac{2}{r^3}\left(e^{-rt_1}-1\right)\right]$$

$$\left.\right]-\left(\frac{1-e^{-v(\mathrm{T}-\mathrm{t}_1)}}{v}\right)\left[\left(\frac{a-bp}{r}\right)\left(e^{-rt_1}-1\right)\right.$$

$$\left.+c\left[\frac{t_1e^{-rt_1}}{r}+\frac{\left(e^{-rt_1}-1\right)}{r}\right]\right]\Bigg]\Bigg\}$$

Furthermore, following M, there are no positive stocks .So

$IC_2 = 0$

(3.18) $$TC_2(\mathrm{t}_1,\mathrm{T})=\frac{1}{T}\left[A+C_1\left\{(\mathrm{a-bp})\frac{t_1^{\,2}}{2}+\left[(\mathrm{a-bp})\,\mathrm{r}+2\,\mathrm{c}\right]\frac{t_1^{\,3}}{6}\right.\right.$$

$$+\left[2\theta(\mathrm{a-bp})-3\,\mathrm{cr}\right]\frac{t_1^{\,4}}{24}+\left[4c\theta-3r\theta(\mathrm{a-bp})\right]$$

$$\left.\frac{t_1^{\,5}}{60}-\left[(\mathrm{a-bp})\theta^2+3cr\theta\right]\frac{t_1^{\,6}}{72}-\frac{c\theta^2t_1^{\,7}}{84}\right\}+C_2\left[\frac{e^{-vt}}{v(\mathrm{v-r})}\left[(\mathrm{a-bp})-\frac{c(2v-\mathrm{r})}{v(v-\mathrm{r})}\right]\right.$$

$$\left[e^{(v-\mathrm{r})\mathrm{T}}-\mathrm{e}^{(v-\mathrm{r})\mathrm{t}_1}\right]+\frac{e^{v(\mathrm{t}_1-T)}}{vr}\left[(\mathrm{a-bp})\right.$$

$$+\frac{c(v\,t_1-1)}{v}\Big]\Big]\left[e^{-rT}-e^{-rt_1}\right]+\frac{ce^{-vT}}{v(\mathrm{v-r})}\Big[$$

$$Te^{(v-\mathrm{r})\mathrm{T}}-\mathrm{t}_1\mathrm{e}^{(v-\mathrm{r})\mathrm{t}_1}\Big]\Big]+C_3\left\{\left(\frac{a-bp}{r}+\frac{c}{r^2}\right)\right.$$

$$\left(e^{-rt_1}-e^{-rT}\right)+\left(\frac{a-bp}{v-r}-\frac{c}{(v-r)^2}\right)$$

$$\left(e^{v(\mathrm{t}_1-\mathrm{T})-\mathrm{rt}_1}-e^{-rT}\right)+\frac{c}{r}(t_1e^{-rt_1}-Te^{-rT})+\frac{ce^{-vT}}{(v-\mathrm{r})}\Big[t_1e^{(v-\mathrm{r})}\,\mathrm{t}_1$$

$$\left.-Te^{(v-\mathrm{r})\mathrm{T}}\Big]\right\}+C\left[(\mathrm{a-bp})\,\mathrm{t}_1+\frac{ct_1^{\;2}}{2}+\frac{(\mathrm{a-bp})\theta\,\mathrm{t}_1^{\;3}}{6}+\frac{c\theta t_1^{\;4}}{8}-\left[\frac{a-bp}{v}\right.\right.$$

$$\left.\left.+c\left(\frac{t_1}{v}-\frac{1}{v^2}\right)\right]e^{v(\mathrm{t}_1-\mathrm{T})}+\left[\frac{a-bp}{v}+c\left(\frac{T}{v}-\frac{1}{v^2}\right)\right]\right]+\frac{c\sigma IccQ^{(\mathrm{n}+1)}\,\mathrm{L}}{2n}$$

$$-(1-\sigma)\,pI_e\left\{\left[\left[(a-bp)\left(\frac{1}{r^2}-\left(\frac{1}{r^2}+\frac{t_1}{r}\right)e^{-rt_1}\right]\right.\right.\right.$$

$$c\left[\frac{t_1^{\;2}e^{-rt_1}}{r}+\frac{2}{r^2}t_1e^{-rt_1}+\frac{2}{r^3}\left(e^{-rt_1}-1\right)\right]\Big]$$

$$\left.\left.-\frac{\left(1-e^{-v(\mathrm{T}-t_1)}\right)}{v}\left[\left(\frac{a-bp}{r}\right)\left(e^{-rt_1}-1\right)+c\left[\frac{t_1e^{-rt_1}}{r}+\left(\frac{e^{-rt_1}-1}{r^2}\right)\right]\right]\right]\right\}\Bigg]$$

4 Numerical Illustration

Utilizing the subsequent values for the different parameters, The developed model has been demonstrated through the consideration of a numerical example.

For case 1 Taking suitable parameter values

A=\$ 1400/ order , C_1=\$ 0.5/ unit ,a=306,b=49.35,p=\$ 7/ unit ,r=.72, c=3, θ=0.75% , C_2=\$5/unit , v =0.7, C_3=\$ 6/unit , σ =0.7, I_{CC} =.09 % / year, n=3, L=0.2 year , I_e =\$ 0.12, I_c =\$ .09, M=0.2 year , C=\$ 5 /unit .

Optimal Result

TC =\$18.1563, t_1=3.50078 ,T =6.88821 .

For Case 2 Taking suitable parameter values:

A = \$1900/ order , C=\$ 5 / unit , C_1=\$ .5 / unit,C_2=\$5 /unit,C_3=\$6 /unit ,a=306, b=49.35, p=\$7 / unit, r=.72, c=3, θ=.75 %, v =.7, σ =.7, I_{CC} =.09 % / year, n=3, L=.2 year, I_e =\$ .12.

Optimal Result

TC =\$18.3885, t_1=4.32139 , T =7.51268 .

5 Convexity

For Case 1

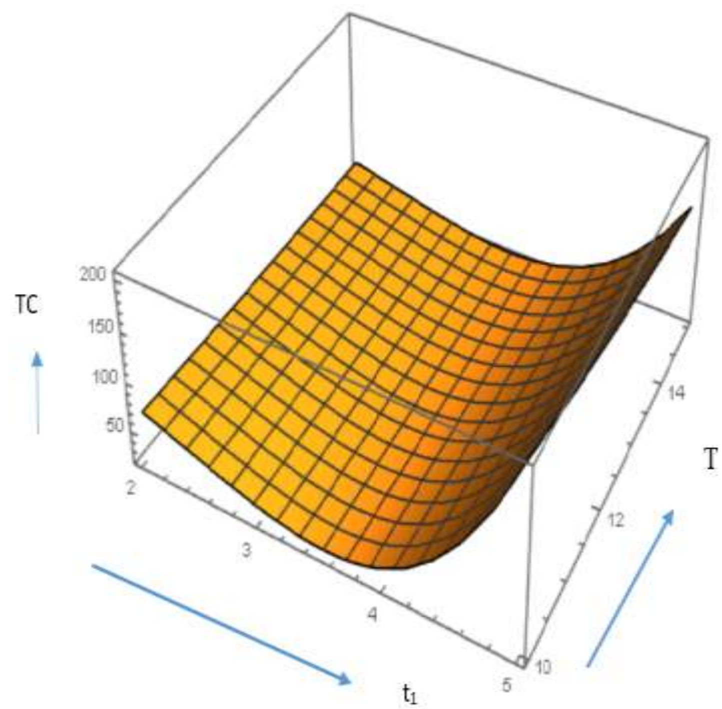

Fig. 5.1 *Represent the convexity in case I w.r.t. T, t_1 and Total cost .*

For case 2

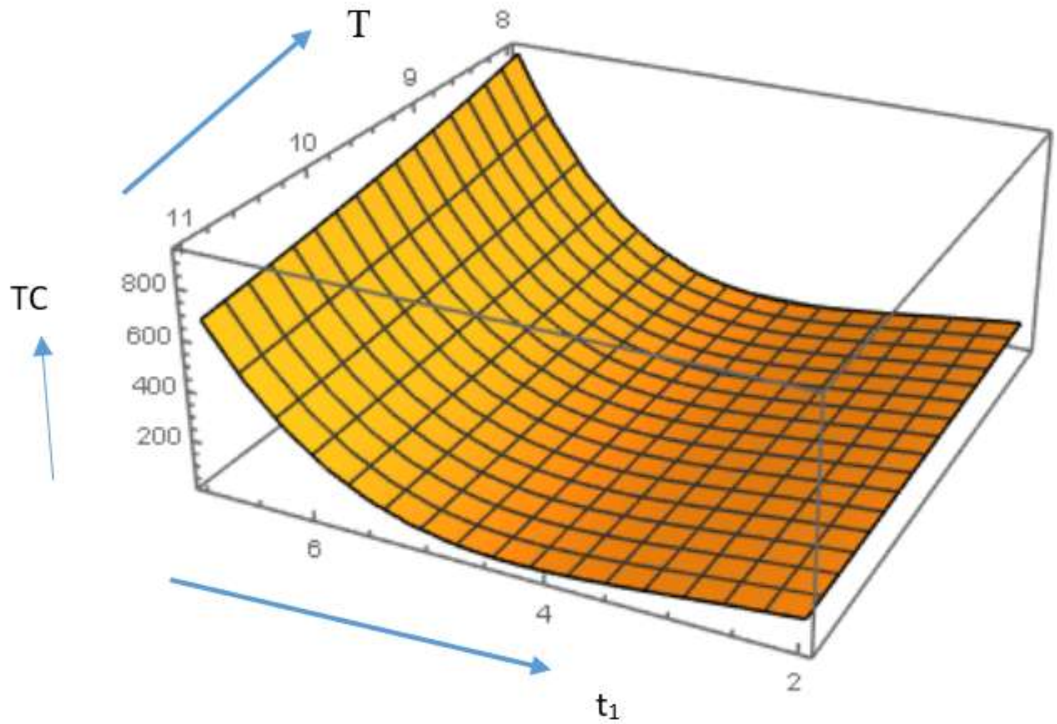

Fig. 5.2 *Represent the convexity in case II w.r.t. t_1, T and Total cost .*

6 Sensitivity analysis

Table 6.1 *For case 1, sensitivity analysis is shown by the following table*

Parameter	%	t_1	T	TC
C_1	20	3.43308	7.0483	19.8427
	10	3.46569	6.97077	19.0807
	-10	3.53869	6.80202	17.0422
	-20	3.57981	6.71345	15.705
C_2	20	3.50085	7.23175	21.4675
	10	3.40138	7.03929	19.934

	-10	3.61355	6.76816	16.0739
	-20	3.74338	6.674	13.5977
C_3	20	3.50095	6.53125	17.8047
	10	3.50089	6.69073	17.9683
	-10	3.50059	7.13931	18.3749
	-20	3.50073	6.96496	18.2254
θ	20	3.31272	7.23175	21.4675
	10	3.40138	7.03929	19.934
	-10	3.61355	6.76816	16.0739
	-20	3.74338	6.674	13.5977
c	20	3.51893	7.268	25.7986
	10	3.50957	7.06429	21.9983
	-10	3.49249	6.73219	14.2753
	-20	3.48464	6.59136	10.3574
σ	20	3.50012	6.67634	15.7446
	10	3.50046	6.77529	16.9592
	-10	3.50108	7.01936	19.3338
	-20	3.50134	7.17531	20.4889
I_{CC}	20	3.50153	6.86605	17.9444
	10	3.50115	6.87706	18.0504
	-10	3.50041	6.8995	18.2619
	-20	3.50004	6.91093	18.3674
n	20	3.50063	6.8929	18.2003
	10	3.5007	6.89076	18.1803
	-10	3.50088	6.8851	18.1269
	-20	3.50101	6.88123	18.0902
L	20	3.50153	6.86605	17.9444
	10	3.50115	6.87706	18.0504
	-10	3.50041	6.8995	18.2619
	-20	3.50004	6.91093	18.3674
I_e	20	3.50136	6.98841	19.0776
	10	3.50107	6.93677	18.6185
	-10	3.50049	6.84241	17.6911
	-20	3.50019	6.79909	17.2231
I_c	20	3.40756	8.57039	24.8187
	10	3.45283	7.39173	21.7872
	-10	3.55181	6.56195	13.9621
	-20	3.60633	6.32029	9.17316
M	20	3.49857	6.95452	18.7564
	10	3.49967	6.92116	18.4608

	-10	3.50192	6.85567	17.8429
	-20	3.50307	6.82356	17.5205

Table 6.2 *For case 2, sensitivity analysis is shown by the following table*

r	20	4.23449	10.3085	25.5591
	10	4.27673	7.86367	22.1806
	-10	4.35915	6.53782	12.8865
	-20	4.40182	6.14158	6.75178
c	20	4.35802	8.35295	28.1693
	10	4.33884	7.84606	23.3513
	-10	4.30539	7.26213	13.3109
	-20	4.29061	7.0605	8.13652
θ	20	4.07592	8.07785	22.8038
	10	4.19115	7.73989	20.8111
	-10	4.47062	7.3518	15.4166
	-20	4.64457	7.23962	11.7086
C_2	20	4.32142	7.51584	18.3919
	10	4.32141	7.51426	18.3902
	-10	4.32137	7.5111	18.3868
	-20	4.32136	7.50952	18.3851
C_3	20	4.32107	7.19396	18.0606
	10	4.32123	7.33558	18.2115
	-10	4.32154	7.74111	18.5996
	-20	4.32166	8.04731	18.8563
σ	20	4.32068	7.30895	16.0588
	10	4.32105	7.40387	17.2309
	-10	4.32171	7.6397	19.5297
	-20	4.32237	8.07573	22.2595
I_{CC}	20	4.32231	7.48113	18.0951
	10	4.32185	7.49676	18.21
	-10	4.32093	7.5289	18.5367
	-20	4.32047	7.54541	18.6846
n	20	4.3212	7.5194	18.4222
	10	4.32129	7.51634	18.3473
	-10	4.32152	7.50823	18.3473
	-20	4.32168	7.5027	18.2957

L	20	4.32231	7.48113	18.0911
	10	4.32185	7.49676	18.24
	-10	4.32093	7.5289	18.5367
	-20	4.32047	7.54541	18.6846
I_e	20	4.32206	7.60465	19.2422
	10	4.32173	7.55726	18.8166
	-10	4.32206	7.60465	19.2422
	-20	4.32071	7.43082	17.5258

7 . Observations

7.1 *Observation from case 1 table*

1. After C_1 increasing, t_1 decreasing and T , Total cost increasing .
2. After C_2 increasing, t_1 fluctuate and T , Total cost increasing .
3. After C_3 increasing, t_1,T and Total cost fluctuate.
4. After θ increasing, $t_1$decreasing and T , Total cost increasing .
5. After c increasing, t_1 , T and Total cost increasing .
6. After σ increasing,t_1 , T and Total cost decreasing .
7. After I_{cc} increasing, $t_1$increasing ,and T , Total cost decreasing .
8. After n increasing, $t_1$decreasing ,and T , Total cost increasing .
9. After L increasing, $t_1$increasing ,and T , Total cost decreasing .
10. After I_e increasing, t_1 , T and Total cost increasing .
11. After I_c increasing, $t_1$decreasing , Total cost increasing and T fluctuate .
12. After M increasing, $t_1$decreasing ,and T , Total cost increasing .

7.2 Observation from case 2 table

1. After r increasing, t_1 decreasing and T , Total cost increasing .
2. After c increasing, t_1 , T and Total cost increasing .
3. After θ increasing, t_1 decreasing and T , Total cost increasing .
4. After C_2 increasing, t_1 , T and Total cost increasing .
5. After C_3 increasing, t_1 ,T and Total cost decreasing.
6. After σ increasing,t_1 , T and Total cost decreasing .
7. After I_{cc} increasing, $t_1$increasing ,and T , Total cost decreasing .
8. After n increasing, $t_1$and T , Total cost increasing .

9. After L increasing, $t_1$increasing ,and T , Total cost decreasing .
10. After I_e increasing, t_1 , T and Total cost fluctuate .

8 . Conclusion

A key element of supply management credit strategy that influences each party's overall costs and purchase decisions is how suppliers, retailers, and customers pay one another. This study addresses the business decline and failure associated with COVID-19 and suggests a plan to encourage supply chain economic recovery. A different method is suggested for upholding business relationships through a hybrid payment plan. The supplier in this study requests n installments of prepayment. Furthermore, a retailer receives fixed-rate loans from a financial institution. This study takes inflation, shortages, partial backlogging, demand dependence on time and price, and instantaneous deteriorating items into account. The model's applicability is demonstrated by a numerical illustration with varying trade credit periods. The numerical example demonstrates how trade credit and late payment tactics can be used to lower a retailer's overall costs. A graphic representation of the overall cost function's convexity in relation to the decision variables is provided.

Assuming that customer demand is time- and price-dependent, this study is restricted to single retailers, suppliers, and customers. It could be expanded to include multi-retailer, supplier, and customer scenarios. By considering stock-dependent demand, this work's analysis can be expanded. It is also possible to investigate the impact of preservation techniques on the deterioration rate. Taking into account the factor of carbon emissions (carbon cap policy) in a fuzzy environment to manage uncertainty could be an intriguing way to further this research. The proposed model can be examined by taking into account how the level of greening affects the cost of manufacturing and selling.

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